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SOME PROBLEMS OF UNRELIABILITY AND HAZARD IN TECHNICAL OBJECTS

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1. Introduction

During last years it may be observed the tendency to isolate from the general reliability theory, problems connected with hazard. This part of theory can be called safety theory. These problems are connected with taking into account consequences of failures.
In the most of technical objects, the failures can be divided into two groups. The first group causes dangerous situation, the second one – only less efficient working of the object. As may be seen the consequences of failures are not equivalent. In the first case we can speak about of hazard, and in the second one about inefficiency of these objects.
The safety problems are not fully solve, we can observe it especially in such specific fields as for example air space, nuclear physics, transportation, fire–fighting, mining, where the probability of direct probability of occurrence of an event causing a dangerous situation is very important.

2. Basic definitions

Reliability of an technical object is determined by random variable *X*–as *worthiness*. Safety of the object is determined by random variable *Y*–as *hazard* (occuring of dangerous situation of the object), *X* and *Y* are described in the following way:

$$X = \begin{cases} 1 & \text{- when the technical object is worthy;} \\ 0 & \text{- when the technical object is unworthy;} \end{cases} \quad (1)$$

$$Y = \begin{cases} 1 & \text{- when the technical object is fail-safe;} \\ 0 & \text{- when the technical object is not fail-safe.} \end{cases} \quad (2)$$

Adequate probabilities is given by the formula:

R=P(X=1) - *reliability (dependability)* – probability of non occurrence of an event causing a object out of order;
Q=P(X=0) - *unreliability (independability)* – probability of occurrence of an event causing a object out of order;
S=P(Y=1) - *safety* – probability of non occurrence of an event causing a dangerous situation;
H=P(Y=0) - *hazard* – probability of occurrence of an event causing a dangerous situation.

3. Mathematical model of the "man-machine" object

Relations between the random variables X and Y are defined by the following conditional probabilities:

$$
\begin{aligned}
&P(Y=1/X=1)=S_r\\
&P(Y=0/X=1)=H_r=1-S_r\\
&P(Y=1/X=0)=S_q\\
&P(Y=0/X=0)=H_q=1-S_q
\end{aligned}
\tag{3}
$$

In the above relations:

S_r - *conditional safety* – is a probability of non-occurrence of an event causing a dangerous situation if the object is worthy;

H_r - *conditional hazard first kind* – is a probability of occurrence of an event causing a dangerous situation if the object is worthy;

S_q - *inefficiency* – is a probability of non occurrence of an event causing a dangerous situation if the object is out order – only less efficient working of the object;

H_q - *conditional hazard second kind* – is a probability of occurrence of an event causing a dangerous situation if the object is out order.

Joint probability of two-dimensional random variable is given by:

$$
\begin{aligned}
&P_{sr}=P(Y=1,X=1)\\
&P_{sq}=P(Y=0,X=1)\\
&P_{hr}=P(Y=1,X=0)\\
&P_{hq}=P(Y=0,X=0)
\end{aligned}
\tag{4}
$$

Regarding the above we obtain:

$$S=P(Y=1)=R\,S_r+(1-R)S_q=1-H_q+\left(H_{q-}-H_r\right)R \tag{5}$$

$$H=P(Y=0)=R\,H_r+(1-H_r)\,H_q=H_r+\left(S_r+S_q\right)Q \tag{6}$$

Expected values and variances of random variables *X* and *Y* are given by:

$$EX=P(X=1)=R \tag{7}$$

$$s_x=\sqrt{R(1-R)} \tag{8}$$

$$EY=P(Y=1)=S \tag{9}$$

$$s_y=\sqrt{S(1-S)} \tag{10}$$

Between the expected values of random variables *X* and *Y* the following relationship:

$$EY=EX\left(S_r-S_q\right) \tag{11}$$

Relation between the variance of random variables *Y* and *X* becomes:

$$s_y = s_x = \sqrt{\left[R(1-R)/S(1-S)\right]} \tag{12}$$

Covariance between the random variables *X* and *Y* the following relationship:

$$s_{xy} = E\left[(X-R)(Y-S)\right] = E(XY) - EX\ EY = R(1-R)\left(H_q - H_r\right) \tag{13}$$

Correlation coefficient is by:

$$r_{xy} = s_{xy}/\left(s_x\ s_y\right) = \left(H_q - H_r\right)\sqrt{\left[R(1-R)/S(1-S)\right]} \tag{14}$$

After transforming the above formula we obtain the relationship betwen the safety *S* and the reliability *R* with the given correlation coefficient r_{XY} and probability S_q stated by:

$$S = \frac{(1+2ASq) \pm \sqrt{\left(1+2AS_q\right)^2 - 4(1+A)AS_q^2}}{2(1+A)}\ . \tag{15}$$

where:

$$A = \frac{1-R}{R\, r_{XY}^2}\ .$$

Using formula (15) three graphs in Figure 1 have been done.

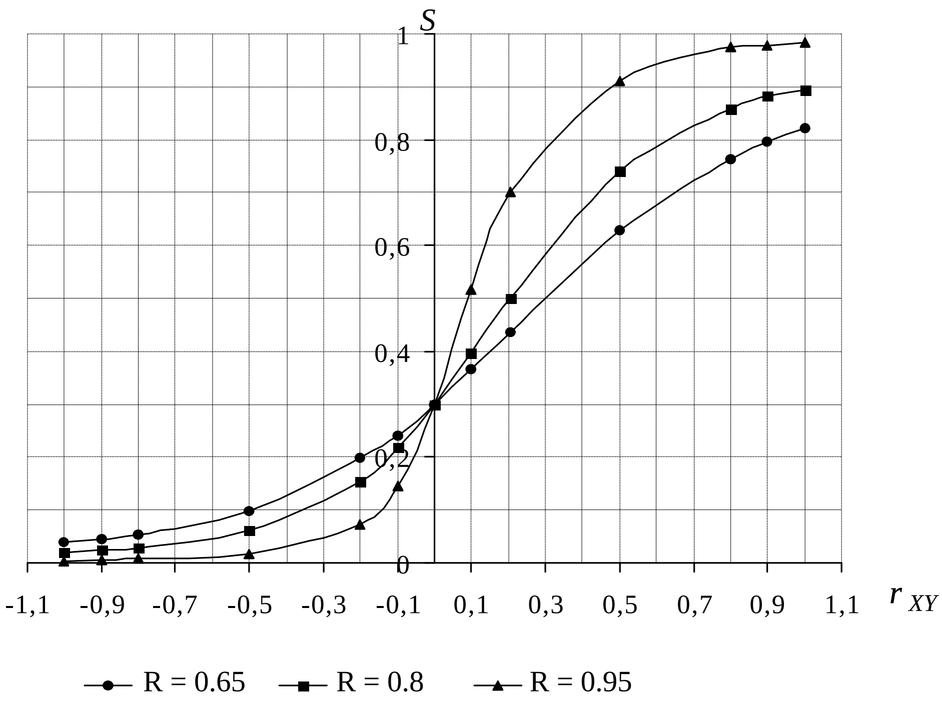

Figure 1. The relationship between the safety and the correlation coefficient
S=f(r_{XY})* for S_q= 0.3, *R = 0.65, 0.8, 0.95

It can be seen from the graph that with the growth of correlation r_{XY} the object safety increases. For $r_{XY} = 1$, $S > R$. It results from this, that some damages do not show negative effects for safety. The function being considered in point $r_{XY} = 0$ is equal to S_q.

For the negative correlation coefficient safety decreases as the reliability increases.

This type of situation results from the reserved objects that are being carried on an technical object increasing reliability while at the some time it creates situation where the safety is decreasing.

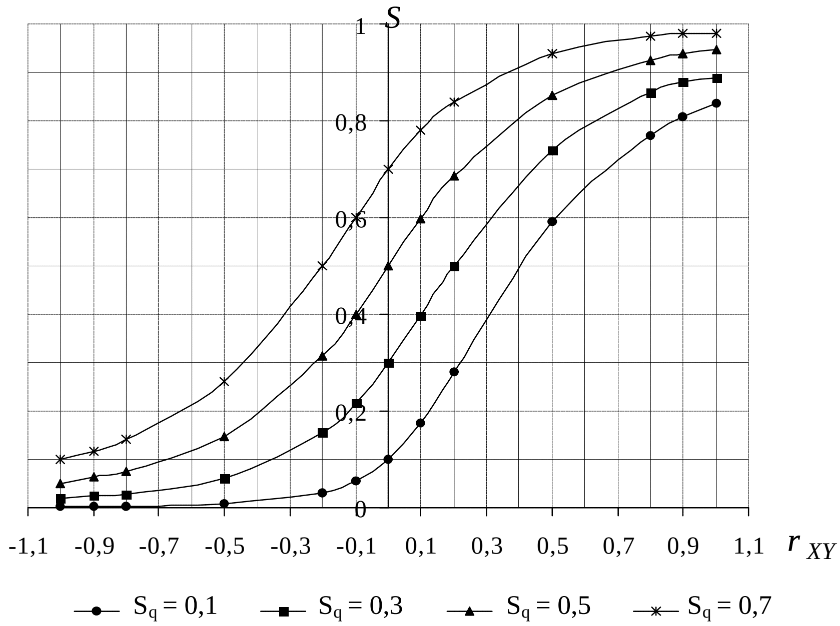

Figure 2. The relationship between the safety and the correlation coefficient
$S=f(r_{XY})$ for R = 0.8 and S_q= 0.1, 0.3 ,0.5, 0.7

With the grown of probability of tolerance of the damage S_q safety of an technical object *S* increases in the function of the correlation coefficient r_{XY}.

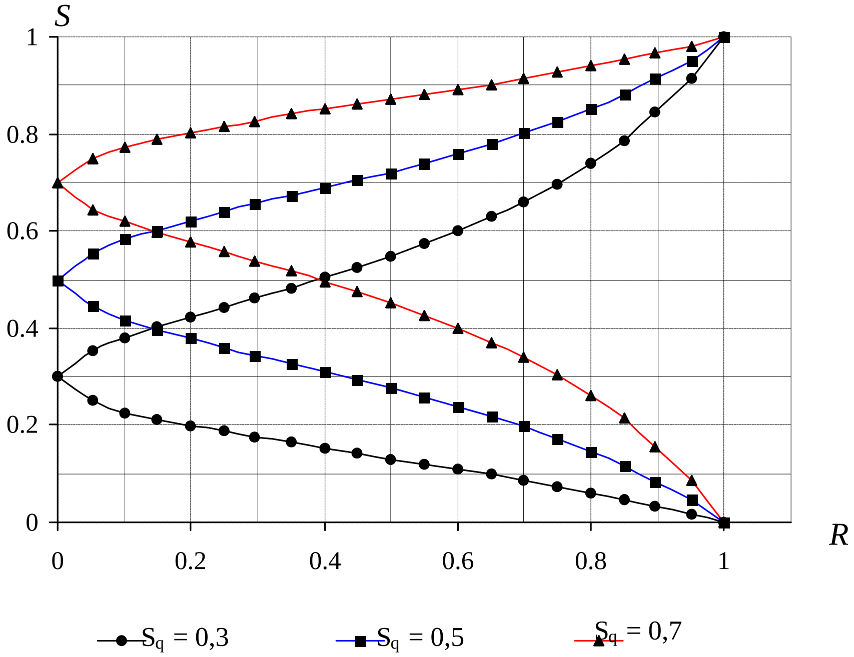

Figure 3. The relation the safety and the reliability
S=f(R) for r_{XY}=0.5 and S_q= 0.3, 0.5, 0.7

The increasing part of the function presents the case when the coefficient of correlation $r_{XY} > 0$, but the decreasing part of the function shows the case when the correlation coefficient $r_{XY} < 0$ is negative. It can be seen the essential influence of the index of tolerance of the damage on the object safety.
For analysis of the problems of the object safety the (5) and (6) relations are useful:
Let us analyse the above relations from the point of view of influence of various factors on the safety *S*.
Safety *S* increases with increase of reliability *R*, if:

a) $H_q > H_r, S_r > S_q, H_r + S_q > 1, r_{XY} > 0$

b) in a special case: $H_r = 0, H_q > 0, r_{XY} > 0$

Safety *S* decreases with increase of reliability *R*, if:

a) $H_q < H_r, S_r < S_q, H_r + S_q > 1, r_{XY} > 0$

b) in a special case: $H_q = 0, H_r > 0, r_{XY} > 0$

Safety *S* is independent of reliability *R*, if:

a) $H_q = H, S_r = S_q, H_r + S_q > 1, r_{XY} > 0$

b) in a special case: $H_r = H_q = 0, r_{XY} = 0$

Hazard *H* may be diminished by increasing inefficiency S_q

4. Recommendations to designers

From the presented above analysis the following recommendations to designers of the reliability and safety objects may be formulated:
In the process of object designing one ought to obtain the approaching zero value of conditional probability H_r of occurrence of an event causing a dangerous situation if the object out of order. It may by reached by choice of such a reliability and safety structure of the object, for which it is practically impossible for an operator to make in exploitation any error causing hazard. As an example, it should be nearly impossible to make a false identification of the actual state of the object
If the condition $H_r \approx 0$, is fulfilled, the safety *S* practically proportional to the object reliability *R*. It means, that it is possible to obtain increase of safety by the increase object reliability. The larger reliability may be obtained by enlarging reliability of object elements or by choice of the object structure with reliability surplus. The most commonly used types of surplus are structural, functional and time surplus.
The safety *S* may be enlarged by enlarging conditional probability S_q of occurrence of inefficiency if the object work incorrectly. The probability S_q depends on the safety structure of the objects. We distinguish the safety structure without surplus (each object failure causes the hazard and with surplus (some of failures cause hazard, other failures cause only inefficiency). One ought to enlarge the inefficiency S_q not changing the object unreliability *Q*. The reliability structure with surplus is in many cases equivalent to safety structure with surplus. The reliability structure without surplus is in most cases the structure with surplus from the point of view of safety, because not each failure causes hazard. The most common forms of safety surplus are functional and time surplus.
Additionaly if the above recommendations are fulfilled, the hazard may reache minimum for the minimal values of object unreliability *Q* and conditional probability H_r of hazard of correctly working object and for the maximal value of object inefficiency.

5. Conclusion

Taking into account the defined notions, correlation coefficient between the random variables *X* – as worthiness and *Y* – *as hazard* was determined in the paper. Analysis of the correlation coefficient enabled us to formulate some recommendations for the constructors of technical objects.
In the paper the formula describing dependency on reliability from safety was considered. Analysis of these formulae permits to state some advice for specialists who investigate the accidents. In the paper the problem was widely considered with respect to the time stress.

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