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PREDICTION OF 3-D CRACK GROWTH IN THIN RIM-GEARS

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1. Introduction

With increased demands for higher performance of gear units the load capacity of gears is a matter of great concern. In order to achieve design goals like reduced weight the rim and web of gears are often design thin. Rims and webs that are to thin can lead to damage problems. Crack type damage can initiate during service or can be generated even during manufacturing processes. If the crack are detected either stages the designer can evaluate the effect of crack on the overall strength of gear.
Classical methods for gear design (e.g. ISO, DIN, AGMA standards) do not take into account parameters related to crack effects. The conventional standards are limited to stress calculation in tooth root and are not applicable to the analysis for thin rim gears. An exception is given in AGMA calculation standard where in conventional approach a stress modifying factor for thin rims is proposed. Not many authors have proposed certain results for effects of crack growth in thin-rim gears and this solutions were limited to 2-D analysis [Lewicki 1997], [Kramberger 2000, 2001].
Performing simulation of influence of gear web arrangements on stresses requires 3-D analysis [Opalic]. The objective of this study is to evaluate the effect of different web arrangements on crack propagation path of initial cracks in thin rim gears. It is assumed that initial crack is initiated at the point, in actual fact at the line of the maximum principal stress in the tooth root.
Computer program FRANC3D [Franc3D 1998] was used to determine stress distributions and model crack propagation. This program uses boundary element modelling technique and principles of linear elastic fracture mechanics to analyse of structures with cracks. The remaining service life is approximately determined with the aid of numerical integration of the Paris law.

2. Method of analysis

Crack growth simulation is the process of crack growth prediction in a structure through time or with increasing load. Performing an engineering simulation of a realistic 3-D structure with crack is difficult. However, the physics and mathematics of arbitrary 3-D crack propagation is not understood very well [Aliabadi 1991]. Very often the analysis requires that certain simplifications must be made, e.g. linear-elastic material behaviour.
In recent years, linear-elastic fracture mechanics (LEFM) has often been applied to evaluating the strength of structures containing cracks. Numerical simulation of crack growth provides a powerful predictive tool to use during the design phase as well as for evaluating the behaviour of existing cracks. In order to simulate crack growth an incremental type analysis is used where knowledge of both the direction and size of the crack increment extension are necessary. For each increment of crack extension, a stress analysis is performed and the stress intensity factors (SIF) at crack front are evaluated. The incremental direction and size along the crack front for the next extension are

determined by fracture mechanics criteria involving SIF as the prime parameters. The area around crack front must be remeshed and the next stress analysis is carried out for the new configuration. The described strategy for 3-D crack modelling is incorporated in the Franc3D analysis programme, which was used to perform the presented analysis [Franc3D 1998].

3. Analysis

3.1 Gear modelling

The cylindrical gears, used as an example for the calculation, were spur gears of a truck gearbox [Flašker 1993]. Table 1 shows basic gear geometry parameters adopted for the calculation. The material properties used in this case were that of steel 42CrMo4 (modulus of elasticity 210000 MPa and Poisson's ratio 0.3).

Table 1. Basic gear geometry parameters

Profile	Involute
Number of teeth on gear	39
Normal pressure angle	24°
Module	4,5 mm
Whole tooth depth	10,41 mm
Addendum modification factor	0,0593
Outside diameter	184,7 mm
Tooth and rim width	28 mm
Rim thickness	6,75 mm
Web thickness	7 mm
Hub diameter	100 mm

Three different cases were considered: a no-web, web in the middle position of the tooth face width and web at one side (Figure 1). Rim thickness was chosen equal 1,5m (m represents module). This thickness is estimated as transition value for given geometry where may occur two different failure mode: tooth or rim breakage.

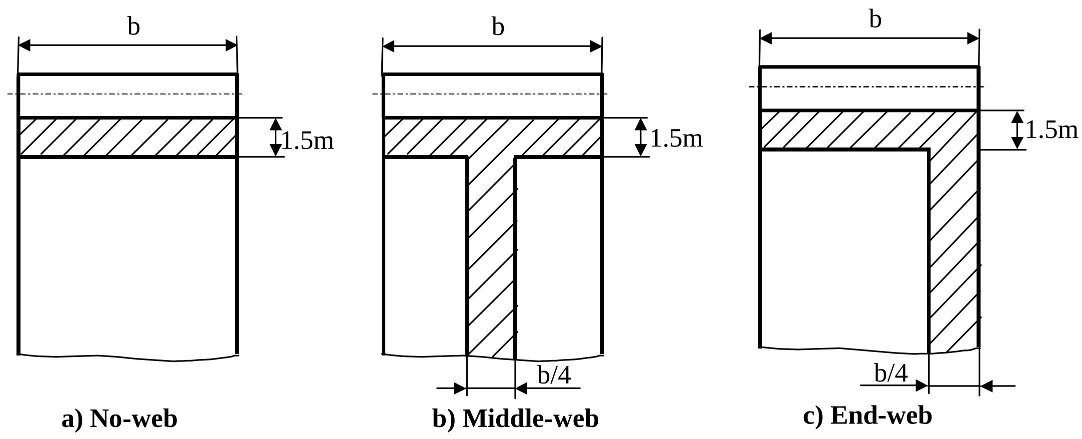


Figure 1. Different web arrangements

The geometrical models of gear used in the analysis are shown in Figure 2. All the models were composed of gear segment with three teeth. The angle between the both end of the model is about 45° corresponding to five teeth. Boundary conditions were applied on the model geometry faces. The both ends of the models were constrained to have zero displacement as shown in Figure 2. The tooth was loaded with normal force F=1737 N/mm, with reference to the engagement at the highest point of single tooth contact, usually assumed to be critical for bending fatigue. Uniform distribution along tooth width was prescribed.

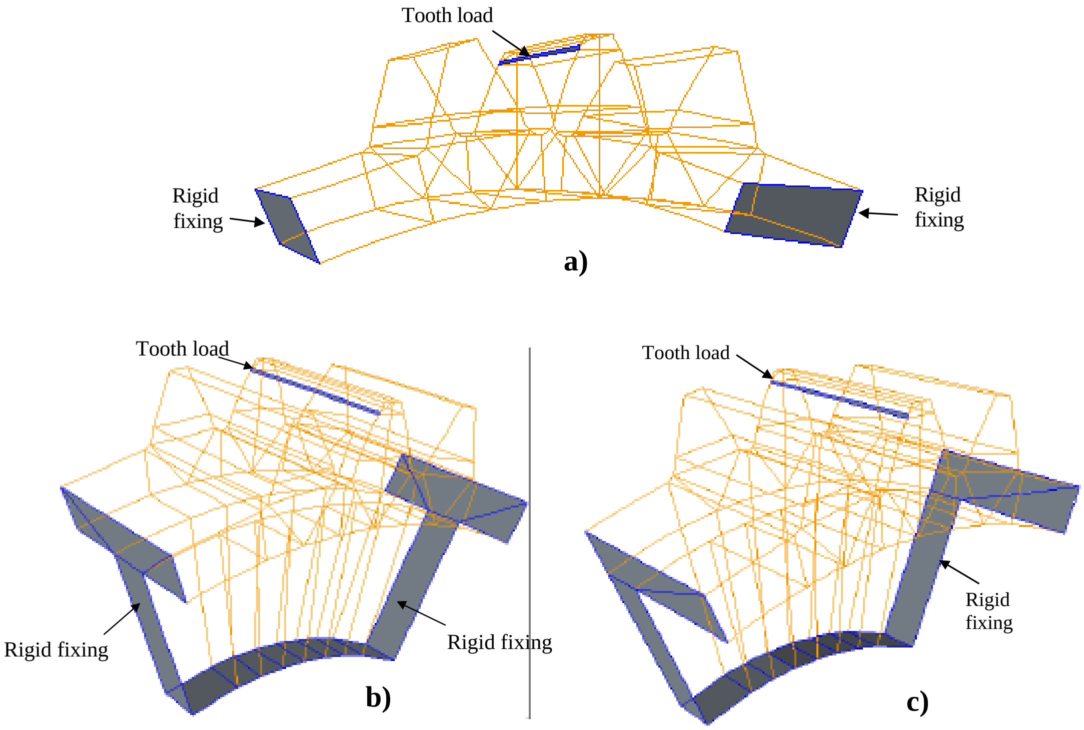


Figure 2. Models of gear segment: a) No-web, b) Middle-web and c) End-web

The gear models were meshed by triangular and quadrilateral surface boundary elements. The mesh was refined in the region of the loaded tooth for improved accuracy. The models shown in Figure 3, have a 1214 boundary elements and 816 boundary nodes (Figure 3a) and 1497 elements and 1074 boundary nodes (Figure 3b), respectively. A similar mesh pattern, as shown in Figure 3b, was created for model with end-web.

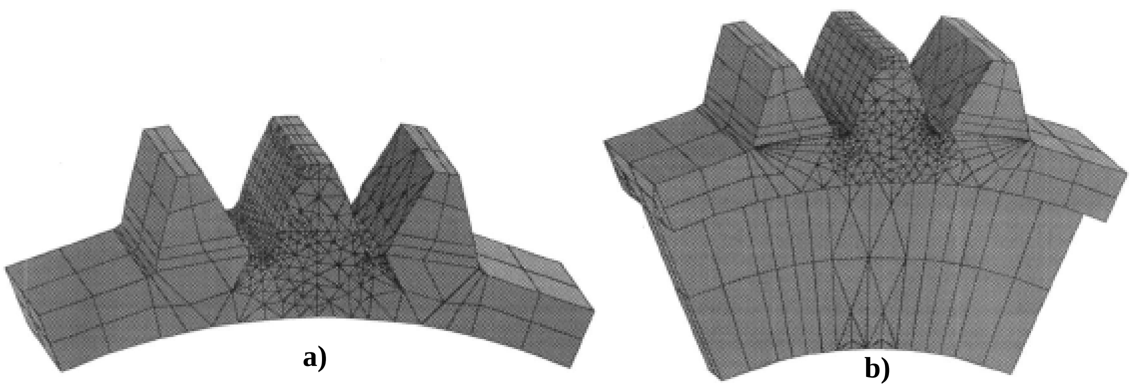
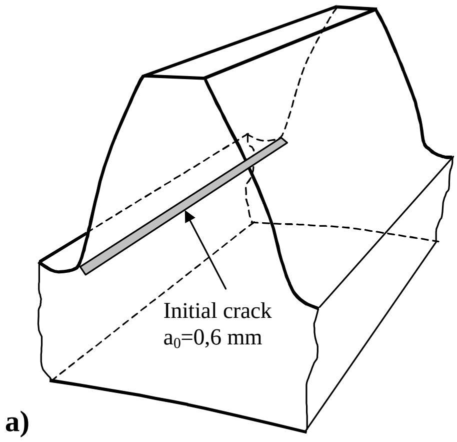


Figure 3. Boundary element mesh of gear model: a) No-web, b) Middle-web

3.2 Crack propagation simulation

First step in performing crack propagation simulation was the stress analysis of uncracked gear. The purpose of this analysis was to determine location of maximum tensile stresses in the tooth root [Lewicki 1999]. For further analysis was assumed that the initial crack starts from area in tooth root where maximum principal stresses occur (an average tangential angle 50°). The initial crack was placed as a part-through crack, as shown in Figure 4a. It was assumed that initial crack already propagated through the brittle case-hardened layer. The length of initial crack was set equal to 0.6 mm, what approximately corresponds to the thickness of case-hardened layer.

The original boundary element model was remeshed at the cracked tooth, as is shown in Figure 4b. Crack growth process was simulated with an incremental crack-extension analysis. For crack growth nine simulation steps were simulated. Analysis required that for each increment of crack extension, a stress analysis was carried out and the stress intensity factors were evaluated at control points at crack front. Maximum extension size at each increment was defined at 0,25 and 0,5mm, respectively. For the modelling of extended crack front points a third-order polynomial fitting using a least square was performed. After remeshing the model was rerun to obtain new solution. The previously described procedure was performed manually within Franc3D system.



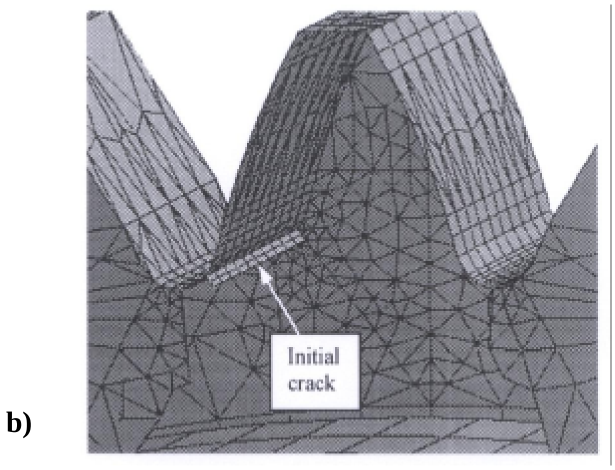


Figure 4. Cracked tooth: a) initial part-through crack in tooth root, b) boundary element mesh at cracked tooth (front elements are removed)

The all three mode stress intensity factors (K_I, K_{II} and K_{III}) were calculated at each propagation step. Figure 5 shows the calculated maximum mode I stress intensity factor at given crack front as a function of crack area for all nine steps of crack growth simulation.

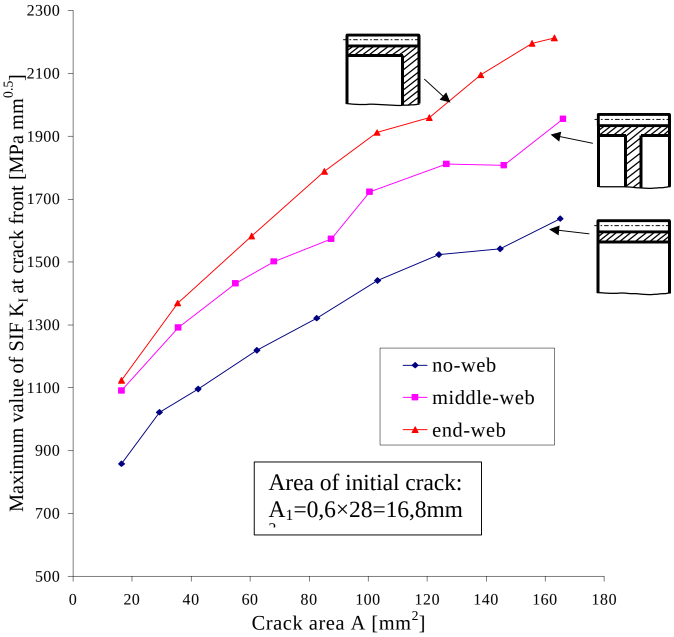


Figure 5. Maximum mode I stress intensity factor at crack fronts versus crack area

Figure 6 shows the extended crack geometry after nine steps of simulation for two various web arrangements. For design without web and the middle-web is resemblance between them. The maximum extension occurred in the middle of crack front (Figure 6a). The crack growth is quite uniform. Predicted failure mode is a tooth breakage. For case with end-web it is evidently that crack propagates non-symmetric with maximum extension at web side (Figure 6b). From this it is clear that this design can lead to rim breakage at non web side.

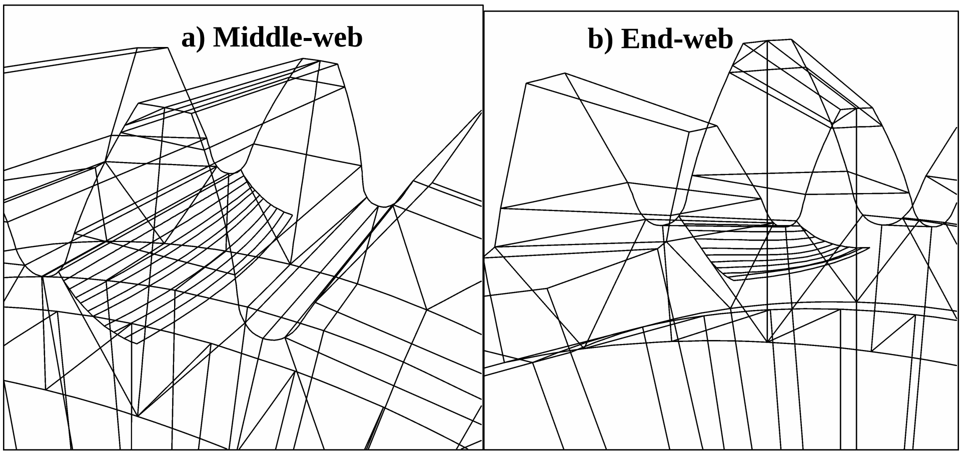


Figure 6. Simulated crack propagation paths for various web arrangements

Finally, the fatigue crack growth using the Paris model was predicted. The Paris fatigue crack growth model correlate crack growth rate and stress intensity range

$$\frac{da}{dN} = C(\Delta K)^n \, . \tag{1}$$

In this equation stress intensity range ΔK was taken as range of maximum value of mode I (ΔK_I), shown in Figure 5. The following material properties were used: C= 4.202x10^{-17} mm/cyc/(MPa√mm)^n and n=4.144, respectively. Fracture toughness value was K_{IC}=2500 MPa√mm. The predicted number of crack propagation cycles is given in Figure 7.

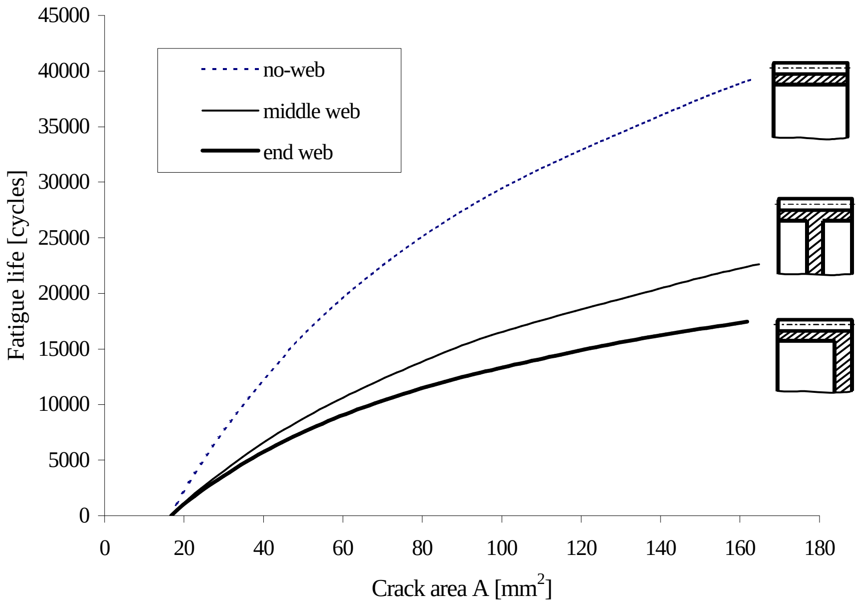


Figure 7. Predicted gear tooth crack propagation life

The highest life occurred for no-web arrangement between supports since it had the lowest stress intensity factors. For cases where rim is reinforced with web it is evidently that crack propagate faster than in no-web case.

4. Conclusion

In the present paper numerical 3-D analysis of gear with complex geometry was performed. The growth of initial through-part crack in tooth root of thin rim gear was simulated. Comparison between gears with presence of web and no-web is done in order to determine the effect of different web arrangements on crack propagation path and life. A crack propagation was simulated using boundary element method with principles of linear elastic fracture mechanics.

The results of undertaken simulation show that for gear rim reinforcement with web predicted crack would propagate faster than for not reinforced gear rim. An asymmetric type of web gear is more disadvantageous compared to symmetric type because it can cause more dangerous fracture mode. In order to validate the numerical results, an experimental work should be performed.

The application shows that 3-D numerical approach can be a very effective tool for the investigation of thin-rimed gears. Furthermore, the advantage of such analysis is that we can analyse real geometry that is not included in conventional standard procedure for gear design. Based on the results the recommended web arrangements of thin-rim spur gear is indicated. With this results and with performing further calculations with different rim thickness design guidelines for thin rim gears and web can be defined.

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