Integration of manufacturing constraints into multi-material topology optimization

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Abstract

Multi-material topology optimization (MMTO) offers lightweight design potential compared to single-material topology optimization (SMTO). For integrating restrictions of manufacturing processes into a MMTO ordered SIMP model, a mapping scheme is used to assign them only to the relevant material sections. By filtering and projection of the design variables, manufacturing constraints are implemented without considerable increase in optimization effort. Optimization results are compared by objective function values, convergence and effects of manufacturing constraints. The improved manufacturability of the optimization results can be achieved with only slight reduction in the objective function value, which is due to the restriction of the design space introduced by manufacturing constraints.

Keywords

Topology optimization, Multi-material, Ordered SIMP, Manufacturability, Heaviside projection method

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Motivation

Topology optimization (TO) is a simulative tool that aims to find an optimal material distribution to achieve one or more optimization goals. Common objectives are the minimization of mass or compliance, usually taking into account part functionality in terms of strength [1]. The most widespread method is SMTO, which uses one material that can be distributed in the available design space to solve the material distribution problem. The Solid Isotropic Material with Penalization (SIMP) approach [2] is often used for optimization. It favors a clear distribution of solid material and voids by penalizing the formation of intermediate densities. The use of a single material limits the solution space right from the beginning and does not allow the use of multi-material designs that are already common today [3]. MMTO methods already exist on an academic scale and can be advantageous compared to SMTO if strength and cost constraints are considered in the optimization [4, 5]. For example, materials with a poor stiffness-to-density ratio (E/ρ) and a higher cost-to-density ratio can be preferably arranged in the design space when using an MMTO algorithm [5]. To improve the direct applicability of MMTO optimization results, it is necessary to consider manufacturing restrictions already during the optimization in order to minimize the rework required on the optimization results and to avoid the resulting deterioration of the objective function value.

State of research on multi-material optimization with manufacturing constraints

Based on the homogenization method presented in [6], the following approaches to solving the material distribution problem in the SMTO have been developed: density, level set, topological derivative, phase field, evolutionary and other approaches [7]. There are density-based and level set approaches for MMTO [8]. Density-based MMTO methods can be differentiated according to their number of design variables. The element stacking method [9] and discrete material optimization [10] use one design variable per element and material. The ordered SIMP model [5] only uses one design variable per element that is taken into consideration for determining the material of an element. For this purpose, the SMTO SIMP method is set up for several density ranges, whereby the densities of the materials are normalized and sorted in ascending order. The advantage of this approach is the easy integrability into existing academic SMTO optimization algorithms as well as the comparably low number of design variables [5]. On the downside, the appearance of checkerboard patterns at the interface between different material phases is to mention [5]. Additionally, elements can assume different materials during the optimization process, which complicates the integration of material-specific manufacturing restrictions.

If the design variables are used exclusively to determine the optimal design, not manufacturable results can be generated. To avoid this, the geometry of the resulting structures can be influenced during the optimization process by the following methods. A common technique is the use of filter functions for the introduction of restrictions in SMTO optimization [11]. Heaviside functions can be used to minimize intermediate densities in the optimization results and to integrate manufacturing restrictions [11, 12]. An alternative projection method are Q-norm projections, which make use of the maximum and minimum operators in the continuous form [13]. Constraints are used in the optimization formulation for a direct restriction of the parameter space of an optimization [14]. Other methods exist such as the geometry projection method, which only allows structures with a specific, fixed geometry to be used in the TO results [15].

Filters are used for manipulating the design variables such that they are mapped onto element space by creating combinations of the input values [16]. All morphological filters have neighborhood sets N^e that contain members of the design variables $\boldsymbol{\Phi}$ for calculating the filtered element density of the current element μ^e. The design variables in a neighborhood set are determined by using the distance between their coordinates $\boldsymbol{x}_i$ and those of the element under consideration $\boldsymbol{x}_{\mathrm{e}}$. Possible forms of the neighborhood set are radial [11], linear or a combination of both [17] known as unidirectional neighborhood set. The values of the design variables Φ_{i} within a neighborhood set are weighted using weighting functions $w(\boldsymbol{x}_{\mathrm{i}},\boldsymbol{x}_{\mathrm{e}})$ and result in the filtered density value of the element under consideration according to equation 1. Depending on the desired influence of the design variables in the neighborhood set, linear [12], cone-shaped [11], uniform or shift-inverse weighting functions [17] can be used.

$$\mu^{\mathrm{e}}(\boldsymbol{\Phi}) = \frac{\sum_{i\in N^{\mathrm{e}}} w(\boldsymbol{x}_{\mathrm{i}},\boldsymbol{x}_{\mathrm{e}})\Phi_{\mathrm{i}}}{\sum_{i\in N^{\mathrm{e}}} w(\boldsymbol{x}_{\mathrm{i}},\boldsymbol{x}_{\mathrm{e}})} \quad (1)$$

To avoid intermediate densities, the Heaviside Projection Method [12] is used to convert a continuous into a discrete design variable field. The continuously differentiable approximation of the Heaviside function in equation 2 is used as the projection function in order to enable the use of gradient-based optimization methods such as the method of moving asymptotes (MMA) [18]. The slope of the Heaviside function is changed using the projection parameter β. $\mu^{\mathrm{e}}(\boldsymbol{\Phi})$ are the filtered design variable values of the element and μ_{max} is the maximum filtered design variable value, that is usually equal to the maximum design variable value Φ_{max}.

$$\rho_{\mathrm{s}}^{\mathrm{e}} = H(\mu^{\mathrm{e}}(\boldsymbol{\Phi})) = \begin{Bmatrix} 1 \text{ für } \mu^{\mathrm{e}}(\boldsymbol{\Phi})>0 \\ 0 \text{ für } \mu^{\mathrm{e}}(\boldsymbol{\Phi})=0 \end{Bmatrix} = 1\text{-}\mathrm{e}^{-\beta\,\mu^{\mathrm{e}}(\boldsymbol{\Phi})} + \frac{\mu^{\mathrm{e}}(\boldsymbol{\Phi})}{\mu_{\mathrm{max}}}\mathrm{e}^{-\beta\mu_{\mathrm{max}}} \quad (2)$$

In a filter operation, all design variables of a neighborhood set have an influence on the element under consideration. Through a subsequent projection, each member of the neighborhood set has the potential to set the element density of the element under consideration to zero or one. A solid projection function applies a restriction to the solid phase of the SIMP approach, whereas a void projection function restricts the elements of the void phase by modifying equation 2 to $\rho_{\mathrm{v}}^{\mathrm{e}} = 1 - H(\mu^{\mathrm{e}}(\boldsymbol{\Phi}))$ [16]. In case of the solid projection function, the element density of the element under consideration is one as soon as at least one design variable in the neighborhood set is greater than zero. This property makes combined filtering and projection steps suitable for integrating manufacturing restrictions into topology optimization. [11]

Although topology optimization results represent an optimal result with regard to the objective function value, the manufacturability of the designs is not taken into consideration. For injection molding, a uniform wall thickness with resulting uniform shrinkage behavior and thus little warpage is advantageous regarding manufacturability [3]. For extruded components, a uniform cross-section in the length direction is required [13]. Milling processes require consideration of the tool body in the topology of a component. For millability, each FEM element removed by milling must have a connection in the milling direction to the edge of the component and the connection must not intersect the component [19]. Furthermore, the minimum diameters and radii as well as the maximum permissible curvatures of voids should be adapted to the milling tool [17]. For parts manufactured by turning, a connection between the holes inside the component and the outer edge is required to avoid inclusions and cross-sections have to be axially symmetrical [13].

The generation of uniform and axisymmetric cross-sections can be implemented in the TO using a mapping method [13]. As a constraint not specifically tailored to a manufacturing process, the minimum length scale can be used for improving TO results for several manufacturing processes. The integration in the optimization algorithm can be realized by using Heaviside functions [16] or Q-norm projections [13]. Symmetry restrictions as well as extrusion restrictions are implemented by using Q-norm projections [13]. The milling restriction is applied differently depending on the used optimization algorithm. In a density-based TO, a restriction of the voids is used for the implementation [17]. The avoidance of void inclusions, which is universally applicable for manufacturing processes, can be implemented using the virtual temperature method [20]. In [21], additional constraints in the optimization algorithm are used for the implementation of non-manufacturing-specific design rules in the ordered SIMP model. The disadvantage of this method is the poorer convergence behavior of the optimization algorithm.

Research question

MMTO algorithms offer additional lightweight design potential compared to SMTO algorithms. However, this can only be fully utilized if the resulting designs also meet manufacturability requirements. By integrating manufacturing constraints into topology optimization, development loops in the product development process can be eliminated which results in a reduction in development time [1]. Restrictions tailored to specific manufacturing processes are already taken into account in the SMTO, but not yet in the MMTO. The aim of this paper is therefore the integration of manufacturing constraints into multi-material topology optimization. For this purpose, the following issues and their effects on optimization must be investigated and resolved:

- How can the manufacturing constraints be assigned only to the corresponding material density section?
- Can the combination of several manufacturing constraints on one material density section be achieved by using nonlinear weighting functions?

Used methods and procedures

The ordered SIMP model according to [5] is chosen as the basis for the MMTO and the MMA according to [18] is used as the optimization algorithm. The design variables are located in the element centers, although their position could also be chosen to be at the node coordinates. To include the manufacturing restrictions in the topology optimization algorithm, a filtering and projection of the design variables takes place in each iteration. This restricts the permissible design space right from the beginning to more advantageous optimization results in terms of manufacturability. The process for transforming the design variables to the element densities is shown in Figure 1. A manufacturing restriction has exactly one filter and one projection function.

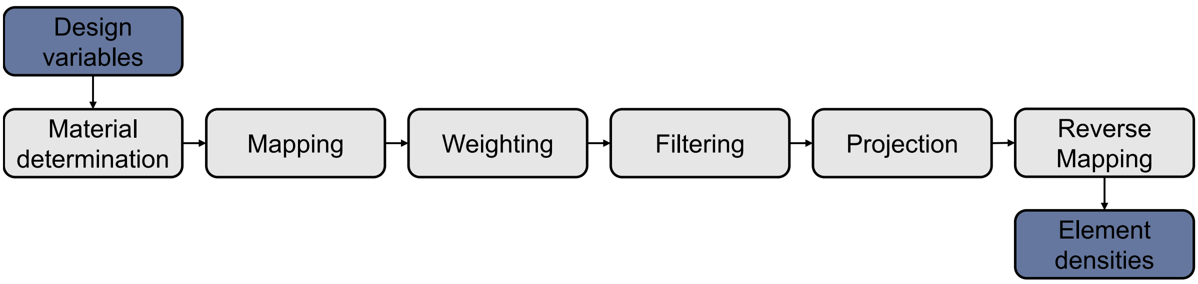


Figure 1: Additional steps in the MMTO optimization algorithm for the integration of manufacturing constraints.

The first step is the **assignment of a discrete material** to each element that is unchanged during an iteration. This is done by evaluating the density region and thus material in which the design variable of the corresponding element in terms of spatial position is located. A **mapping** of all elements of the same material to an SMTO SIMP method is used to ensure that the material assignment of the elements remains the same before and after the application of the manufacturing restrictions, as shown in Figure 2 b). Depending on the phase that should be restricted by the manufacturing constraint, either a solid projection function for the upper phase or a void projection function for the lower phase is used in the constraint formulation. A material located in the center of the ordered SIMP model can be reached from both the upper and lower material sections and is referred to as the duality of the ordered SIMP model in the following. To apply a manufacturing constraint to a material in the middle of the ordered SIMP model, restrictions have to be used for both material sections around the material of interest.

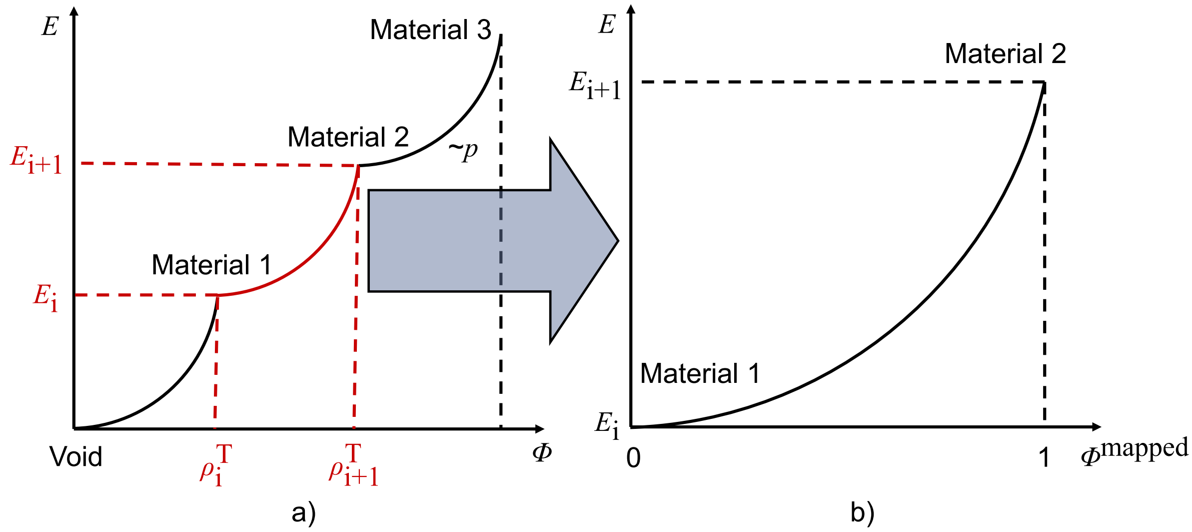


Figure 2: Mapping of a) the ordered SIMP model with three materials to a b) SMTO SIMP method.

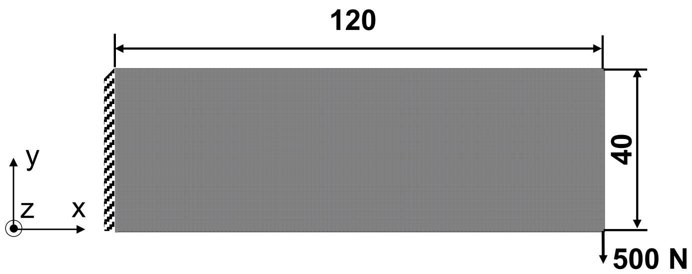
The method presented in [22] is used to include several manufacturing constraints for a material section by filtering the design variables with **nonlinear weighting functions**. If there is just one manufacturing restriction defined for a material, a linear weighting function is used instead of the nonlinear one. The design variables of the elements belonging to other materials are not included in the **filtering** and the subsequent **projection** step. After all filter and projection functions of all manufacturing restrictions have been applied, they are reintegrated into the MMTO ordered SIMP model by means of a **reverse mapping** step. After the reintegration, all elements still have the same material as before the mapping, but may have different element densities than the design variables located in the element centers. The modification of the element densities changes the sensitivity of the objective function and the constraint functions with respect to the design variables. For obtaining the modified sensitivities, the correction is made by multiplication with the terms resulting from the chain rule.

After the convergence of the optimization algorithm, an interpretation of the optimized element densities is required due to the occurrence of intermediate densities in the TO results. To achieve the interpretation, the SIMP curves of the ordered SIMP model are divided in the middle of each material section which leads to a clear material definition for each element.

The **minimum length scale** uses a filter with radial neighborhood set and cone-shaped weighting function. The combination of several manufacturing restrictions on one material is required either by the duality of the ordered SIMP model or several manufacturing restrictions for one material. This can be achieved with the approach presented in [22] by using nonlinear weighting functions.

The **milling restriction** applies a filter with the unidirectional neighborhood set and the shift-inverse weighting function to the design variables in order to obtain the element densities [17]. Due to the local definition of the manufacturing constraints to a material, no elements with a different material are included during the filtering of the design variables. Although this ensures the belonging of an element to the same material throughout an iteration, the influence of elements with other materials on the manufacturing constraint is neglected.

The integration of the minimum length scale and the milling restriction into the ordered SIMP model is investigated using a 2D Messerschmitt-Bölkow-Blohm (MBB) beam with the dimensions 120x40 mm. It has a load application point at the lower right corner with a force of 500 N acting in direction of the negative y-axis and a fixed support on the opposite edge, as shown in Figure 3 a). The meshing in ANSYS WORKBENCH 2023 R2 is done with Solid 185 elements with 1 mm edge length and linear shape functions, which generates a regular mesh with 4800 elements, 9922 nodes and 29766 degrees of freedom. The MBB has only one element layer in thickness direction in order to be able to calculate 2D examples with the 3D implementation of the ordered SIMP model in MATLAB® R2023B. The material models shown in Figure 3 b) are used for the optimization.



a)

Material properties	Aluminum	PP
E in MPa	70000	3600
ρ in g/cm^3	2.7	0.92
G in MPa	26923	1385
ν	0.3	0.3

b)

Figure 3: MBB with fixed support on the left side with a force of 500 N acting upon the bottom right corner; b) material properties of aluminum and polypropylene (PP).

To create the ability of using more than one constraint on a material section of the ordered SIMP model, nonlinear weighting functions with the parameters α_s=0.0002 and α_v=0.0005 are used. The initial element density is set to 0.3 as well as the target volume fraction for the SMTO and MMTO optimizations. The initial penalty factor of two is increased up to a maximum penalty factor of five. The maximum number of iterations is 500. An increment of 50 iterations is used to increase the continuous parameters. The minimum allowable change in density is 0.0004. The minimum element density is set to 10^{-6}, while the maximum design variable value is one. The initial projection parameter starts at one and is increased up to a maximum projection parameter of 50. The filter radius is 2.5 mm.

Results and discussion

The optimization results of the 2D demonstrator are compared based on the objective function value, its convergence and the interpreted relative mass fractions. For their computation, the mass of the optimized structure is set in relation to the mass of the total design space filled with aluminum. The resulting component designs are also assessed with regard to the effects of the manufacturing restrictions. An overview of the results obtained with the investigated demonstrator configurations is listed in Table 1.

Table 1: Overview of the results obtained with the 2D MBB.

Optimization	Manufacturing constraint	Objective function value in mJ	Interpreted relative mass fraction in %
SMTO	none	1856.55	32.33
MMTO	none	2208.23	30.17
SMTO	Milling restriction	21043.74	29.85

MMTO	Minimum length scale	2748.06	29.94
MMTO	Milling restriction	4037.58	34.87
MMTO	Combined	2931.01	30.21

In Figure 4 a), an SMTO with aluminum and without manufacturing constraints is depicted. In comparison with the resembling MMTO result with aluminum and PP in Figure 4 b), both optimizations produce structures with thin struts and local checkerboard patterns. Those undesired patterns are optimization results which are difficult to manufacture or have to be interpreted manually. They are a result of the slight reinforcing effect of the elements possessing the minimum element density, which serve as a bridge between the elements with solid material. The objective function value of the SMTO is lower than that of the MMTO, because no restrictions on component strength and costs are considered in the optimization [5].

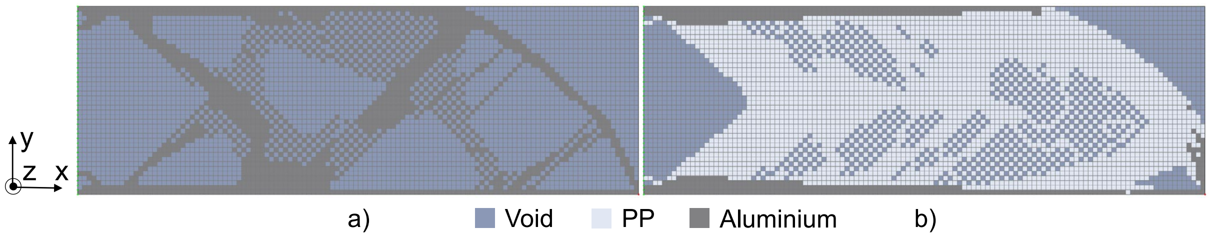


Figure 4: Material distribution of a) SMTO and b) MMTO without manufacturing restrictions.

The MMTO result with minimum length scale on aluminum, PP and void in Figure 5 a) does not contain a checkerboard pattern. The strut thicknesses fulfill the minimum length scale, that is depicted as reference in the figure. The aluminum struts at the edge of the design space are only half the thickness of the minimum length scale, since fewer design variables are in the vicinity of the edge elements. Therefore, the number of design variables in the neighborhood sets of the edge elements is less than of fully surrounded elements in the design space.

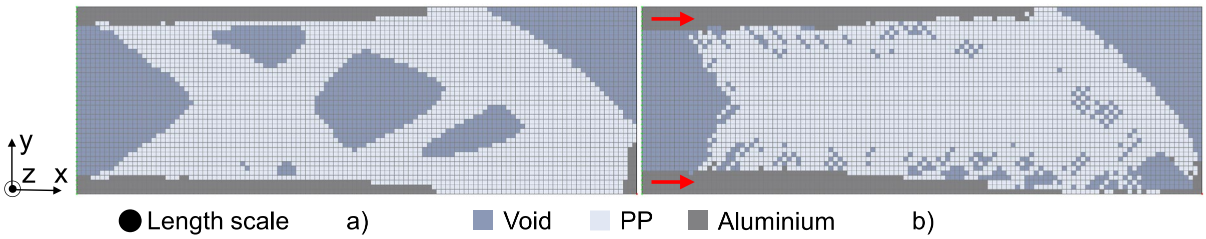


Figure 5: Material distribution of a MMTO with a) minimum length scale on aluminum, PP and void and b) a milling restriction on aluminum.

For the milling restriction on aluminum shown in Figure 5 b), all elements with aluminum as material should have a connection with elements of the same material to the left side of the design space. The restriction leads to a thickening of the aluminum struts compared to the result with the minimum length scale in Figure 5 a). Individual aluminum elements are created around the load application point due to the lack of consideration of elements of other material sections in the milling restriction. As no manufacturing restriction is applied to the PP structure, checkerboard patterns are not prevented and appear in the optimization results.

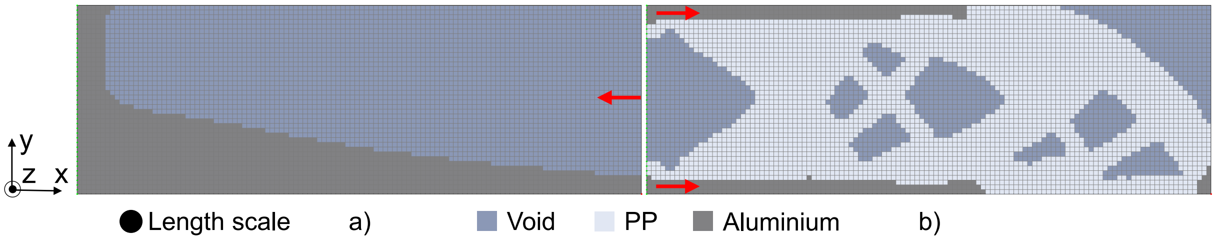


Figure 6: Material distribution of a) a SMTO and b) a MMTO with milling restriction on aluminum and minimum length scale on PP.

The use of a milling restriction for the SMTO with a maximum projection parameter of 200, a maximum penalty factor of 10 and an increment for increasing the continuous parameters of 25 leads to the optimization result depicted in Figure 6 a). Due to the large void region in the upper part of the design space, the part is millable in the direction of the red arrow. The objective function has the highest value in comparison to all other tested configurations due to the low exploitation of the lightweight potential of the TO. An improved behaviour is observed by using a MMTO with a milling constraint on aluminum and a minimum length scale on PP, as shown in Figure 6 b), which produces a manufacturable result with a better objective function value. Compared to the nonrestricted MMTO in Figure 4 b), the objective function value is 33 % higher with a slightly higher relative mass fraction. The optimization result thus leads to improved component manufacturability without the necessity of reworking by product developers, which would lead to an unpredictable change in the objective function value.

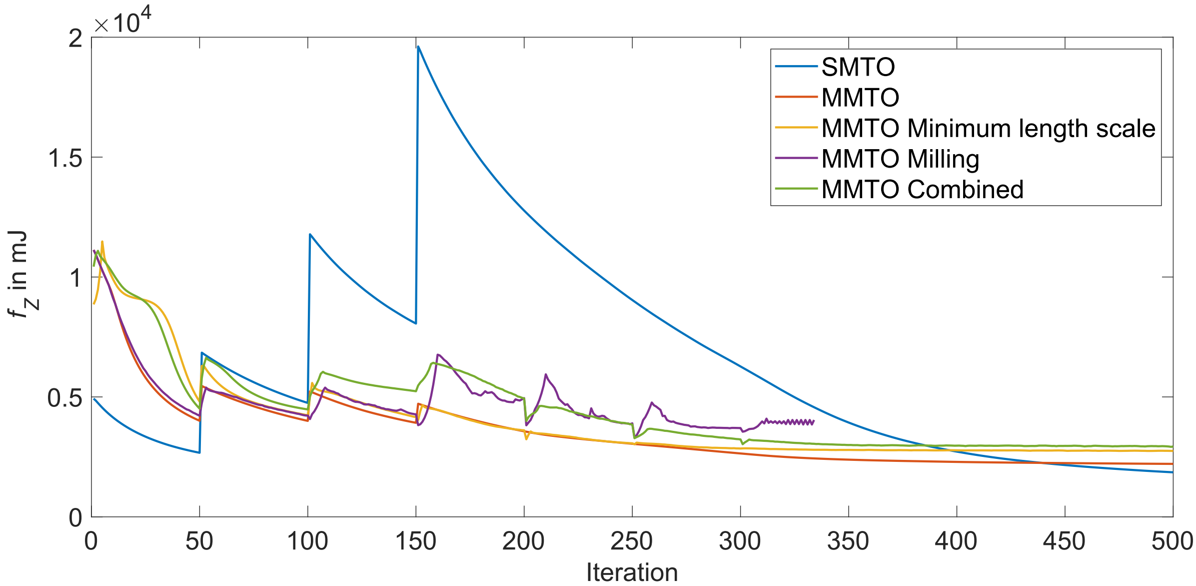


Figure 7: Convergence history of the optimization results shown in Figure 4 to Figure 6 without Figure 6 a).

The convergence history of the optimization with and without manufacturing constraints in Figure 7 shows a deterioration of the objective function value due to the implementation of manufacturing constraints in the TO. When a minimum length scale is applied to all phases of the MMTO, the change of the design variables from iteration 250 onwards is more restricted than in the nonrestricted optimization, which is noticeable in an increased objective function value. Only the optimization with the milling restriction on aluminum ends before reaching the maximum number of iterations and shows an oscillating convergence behavior at the end due to the change of elements between different material sections. A combination of the milling restriction and the minimum length scale converges after 350 iterations, but continuous until the maximum number of iterations is reached. The objective function value for the two optimizations with minimum length scale increases at the beginning of the optimization. Because of the modification of the design variables by filtering and projection, element densities increase. This also affects the mass constraint such that the volume fraction is exceeded. Until the mass constraint is satisfied, the optimization algorithm scales the design variables without modifying the density distribution, which leads to an increase in the objective function value. The same effect is observed at the iteration points when increasing the penalty factor and the projection parameter. Adding more manufacturing constraints to the MMTO leads to an increase in the number of iterations needed to compensate for the parameter increases. The SMTO has a higher sensitivity to the increase in the penalty factor than the MMTO due to the damping effect of the filter and projection functions on the changes in the design variables. The initial values of the objective function differ for the MMTO optimizations due to the change in element densities caused by the manufacturing constraints. In case of the SMTO and MMTO, the difference occurs because of the different scaling factors of the SMTO SIMP method and the MMTO ordered SIMP model.

All in all, when using the implemented manufacturing constraints for the optimization, the speed of convergence is slightly reduced, but the optimization reaches nethertheless solutions with objective function values comparable to those of unconstrained MMTO optimizations.

Conclusion and outlook

Due to the advantageous expansion of the design space through the use of multiple materials, interest in MMTO optimization algorithms is rising. For a more direct usability of MMTO optimization results, the integration of manufacturing constraints is required. Based on the MMTO ordered SIMP model, this paper implements a minimum length scale and a milling constraint by combining filtering and projection methods. The local application of material-specific manufacturing constraints is implemented by means of a mapping procedure. Nonlinear weighting functions enable the application of several manufacturing constraints to a material section. The investigation of the manufacturing constraints on a 2D demonstrator show more feasible optimization results, but also an increase in the objective function value. Further research is necessary with regard to the usage of elements belonging to other materials in the filtering as well as the investigation of the manufacturing constraints for more than two materials and in 3D, as shown in Figure 8. Furthermore, for a better utilization of the lightweight potential of MMTO designs, additional strength and cost constraints should be considered for the optimization.

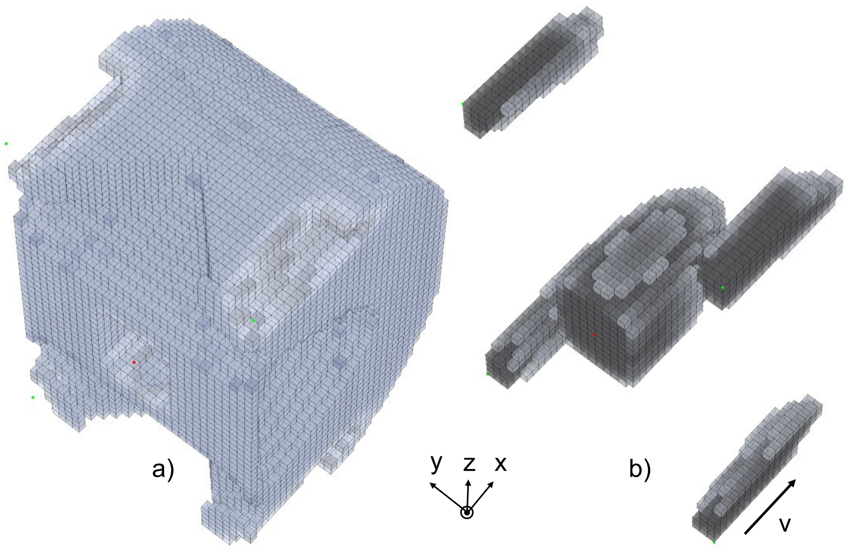


Figure 8: Investigation of the manufacturing constraints on a 3D demonstrator with a) PP and b) aluminum.

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